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Equipment failure rate updating—Bayesian estimation

Ahmad Shafaghi *

ABS Consulting, 16855 Northchase Drive, Houston, TX 77060, United States

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Abstract

The paper presents a Bayes' method for augmenting generic equipment failure data with a prior distribution – predicated on the evidence, e.g., plant data – resulting in a posterior distribution. The depth of the evidence is significant in shaping the characteristics of the posterior distribution. In conditions of insufficient data about the prior distribution or great uncertainty in the generic data sources, we may use "constrained non-informative priors". This representation of the prior preserves the mean value of the failure rate estimate and maintains a broad uncertainty range to accommodate the site-specific event data. Although the methodology and the case study presented in this paper focus on the calculation of a time-based (i.e., failures per unit time) failure rate, based on a Poisson likelihood function and the conjugate gamma distribution, a similar method applies to the calculation of demand failure rates utilizing the binomial likelihood function and its conjugate beta distribution.

Keywords: Bayes' theorem; Failure rate; Prior distribution; Posterior distribution; Poisson distribution; Gamma distribution; Conjugate gamma distribution

1. Introduction

The well-established quantitative risk analysis methodology [1] begins with the task of hazard analysis (e.g., HAZOP) in which potential hazardous events (scenarios) are identified. The scenarios provide the answer to the question of "what could happen?" There are other questions still need responses: "how often?" and "what is the impact?" Each scenario has two dimensions frequency and consequence. The risk of each scenario – to certain population, the environment or the asset – is the product of frequency and severity of consequence associated with the scenario. The total risk posed by the plant is therefore the integration of the scenario risks. Garrick and Kaplan in a classic paper [2] provide a quantitative definition of risk in terms of the idea of a "set of triplets". The definition is extended to include uncertainty and completeness, and the use of Bayes' theorem is described in this connection. The definition is used to discuss the notions of relative risk, and acceptability of risk.

Equipment failure rates are a main ingredient in any risk or reliability analysis. In a risk analysis, the failure data is needed to estimate the frequencies of events contributing to risks posed by a facility. And in a reliability analysis, they are required to predict an unavailability or unreliability of a system. But, the question is where are we going to get the data from? There are two sources of hard data: data collected at a facility – "plant specific" – and data reported by industry – "generic" data. One of the sources of plant specific data is work orders. Unless it is designed for the purpose, work orders are inherently inconsistent and in some cases convoluted.

Now suppose you are conducting a risk assessment study of a plant, which consists of, say, 13 pressure vessels among other equipment items and it has been in operation for 10 years. To dramatize the situation let us assume that the plant has zero number (or any number) of pressure vessel failure (of any kind) since the startup. In other words the plant has zero failure in 130 pressure-vessel-years. Now, is it justifiable to use this information for estimating the risk associated with the pressure vessels at this plant?

Generic data, which are publicly available, e.g., for the chemical process industry [3], for the nuclear industry [4], and for the offshore installation [5], which represent a cross-section of industry. From statistical point of view, most of these sources will provide you with valid point estimates and occasionally lower and upper bound values. However, generic data suffers a major shortcoming, which is non-specific.

Although both approaches provide you with some estimates, neither could produce representative equipment failure frequencies. This is because plant specific data is statistically invalid

* Tel.: +1 281 673 2776.
E-mail address: ashafaghi@absconsulting.com.

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due to a short duration of data collection or limited population of equipment. Generic data, on the other hand, does not reflect the characteristics and conditions of the plant that the equipment is operated under. Hence the use of plant specific or generic data would not to help estimate realistic risk or reliability of a plant.

There is a third way, which is often known as data augmentation, which is performed using the Bayesian methodology. In this approach we use generic data as *a priori* and plant specific data as an evidence (likelihood) to obtain posterior.

2. Bayesian updating methodology

Bayesian statistics is based on the subjective definition of probability as 'degree of belief' and on Bayes' theorem, the basic tool for assigning probabilities to hypotheses combining a prior judgments and experimental information [6].

The approach presented is known as two-step Bayesian methodology [7]. There are many sources of information on Bayes' theory and applications. The two outstanding books by Winkler [8] and Janes [9] are among the best sources of Bayes' theorem. Winker's is on inference and decision making while Janes' book has been the trademark of Bayesian methodology in the risk community.

The approach consists of three main tasks, as follows:

1. Define a prior distribution for the equipment failure rate.
2. Gather evidence, known as the likelihood function.
3. Construct the posterior distribution using Bayes' theorem.

In the context of failure rate estimation, the Bayes' theorem is presented in a functional relationship, as follows:

$$f_{\text{post}}(\lambda) \propto \text{likelihood}(\lambda) f_{\text{prior}}(\lambda) \tag{1A}$$

or

$$f_{\text{post}}(\lambda) \propto Pr(X = x|\lambda) f_{\text{prior}}(\lambda) \tag{1B}$$

where λ = equipment failure rate; $f_{\text{prior}}(\lambda)$ = posterior distribution of failure rate (λ); likelihood (λ) = likelihood function of failure rate (λ); $\Pr(X=x|\lambda)$ = likelihood function as function of λ, for given failure event (x); $f_{\text{post}}(\lambda)$ = posterior distribution of failure rate (λ).

The choice of the prior distribution signifies the analyst's state of knowledge regarding the equipment failure rate. The prior distribution may be derived from a single source, or from a collection of available sources. In failure rate estimation, often generic data is used as the basis for the prior distribution. In case hard data is not available or not from a reputable source, expert opinions may be used to define a prior. Expert opinion is acquired by special techniques such as the Delphi method [10].

Evidence is based on the statistics collected at a specific facility. If the evidence were too limited, then the posterior would resemble the prior. It is significant to remember that the evidence must be independent of the prior. As the historical data becomes larger, several patterns emerge [15]:

- The posterior distribution bears less resemblance to the prior distribution, because the data become the dominant factor.
- The posterior would be narrower and centered around maximum likelihood estimate, implying less uncertainty in the results.

2.1. Choice of prior distribution and likelihood function

Let us first start with the likelihood function, which is best defined by the Poisson distribution defining the behavior of the facility failure data. This is an appropriate distribution for random variables that involve counts or events (such as pressure vessel failure) per unit time.

The Poisson distribution is presented below:

$$Pr(X = x|\lambda) = \frac{e^{-\lambda t}(\lambda t)^x}{x!} \tag{2}$$

where x = number of events (failures); t = time interval.

The conjugate family of prior distributions for Poisson data is the family of gamma distributions. That is to say the uncertainty of the failure rate (λ) is defined by the gamma distribution. The gamma distribution and the event data can be combined to result in another gamma distribution. This is the meaning of the conjugate family.

In the context of Bayes' theorem, when we choose the gamma distribution for the prior, updating it by the Poisson likelihood model, then the posterior distribution is also constructed by the gamma distribution.

The gamma distribution for the prior with two parameters of scale factor (α) and the shape factor (β), which is shown below:

$$f_{\text{prior}}(\lambda) = \frac{\beta^{\alpha}}{(\alpha-1)!}\lambda^{\alpha-1}e^{-\lambda\beta} \tag{3A}$$

Expression (3A) can also be presented as:

$$f_{\text{prior}}(\lambda) \propto \lambda^{\alpha-1}e^{-\lambda\beta} \tag{3B}$$

Note that this expression is valid only when α takes a positive value.

This is known as the gamma (α,β) distribution, where the mean and variance are defined as follows:

$$\text{Mean}\,(\lambda) = \frac{\alpha}{\beta} \tag{4}$$

$$\text{Variance}\,(\lambda) = \frac{\alpha}{\beta^2} \tag{5}$$

2.2. Posterior distribution

Using the gamma distribution for the prior and Poisson for the likelihood, the updated distribution is also a gamma distribution. The resulting posterior distribution, which is a combination of the prior gamma distribution and Poisson distribution for the likelihood function, is also the gamma distribution, given in Eq. (4), which defines conjugate:

$$f_{\text{post}} \propto e^{-\lambda t}\frac{(\lambda t)^x}{x!}\lambda^{\alpha-1}e^{-\lambda\beta} \tag{6A}$$

Simplified further, results in:

$$f_{\text{post}} \propto \lambda^{(x+\alpha)-1} e^{-\lambda(t+\beta)} \tag{6B}$$

The posterior gamma distribution parameters are simply calculated by the following equations:

$$\alpha_{\text{post}} = x + \alpha_{\text{prior}} \tag{7}$$

$$\beta_{\text{post}} = t + \beta_{\text{prior}} \tag{8}$$

Eqs. (7) and (8) along with Eq. (4) (the mean) are the primary tools for the Bayesian update in this study.

Based on Atwood's [17] priors, for Poisson data, the constrained non-informative prior is a gamma distribution with the shape factor and scale factor of the following values:

$$\alpha_{\text{prior}} = \frac{1}{2} \tag{9}$$

$$\beta_{\text{prior}} = \alpha_{\text{prior}}/\text{prior mean} \tag{10A}$$

$$\beta_{\text{prior}} = 1/2\,(\text{prior mean}) \tag{10B}$$

Given the best estimate for mean value of failure rate (from generic data), then the parameters of the gamma prior distribution, α_{prior} and β_{prior}, will be calculated according to Eqs. (9), (10A) and (10B).

3. Case study

The remainder of this paper focuses on the use of the methodology described above to estimate pressure vessel failure rates for several failure modes.

3.1. Prior distribution

Once the prior distribution is known it is required to estimate its parameters, e.g., the mean value. The quality and quantity of generic data as well as the analyst's preference dictate the method of generating the parameters. The analyst may have access to a single or multiple credible data sources. In the multiple source case, the analyst may simply choose to select the most reliable, or employ one of a number of methods to merge these data into a single point estimate. The methods range from calculation of an arithmetic mean to the use of sophisticated Bayesian procedures [11].

For the purpose of this analysis, we have searched various generic data sources applicable to pressure vessels; the results are given in Table 1. The mean values in Table 1 are taken directly from the referenced data sources given in the table. The data includes "disruptive" and "no-disruptive" failure modes. The definitions of the terms used in Table 1 are given below:

- Disruptive failure—"a breaching of the vessel by failure of the shell, head, nozzles or bolting, accompanied by a rapid release of the large volume of the contained pressurized fluid" [12].
- Non-disruptive failure—"a condition of crack growth rate or flaw size that is corrected, and which if it had not been corrected, could have reached a critical size and led to disruptive vessel failure" or "a local degradation of the pressure vessel boundary that is localized cracking with or without minor leakage. Such a crack would not reach critical size and lead to disruptive vessel failure" [12].
- Range factor—the range factor implies the level of confidence that the analyst has in the data source. The smaller the range factor, the higher the confidence of the analyst in the data source. For the gamma distribution, the range factor can be estimated by the square root of the ratio of the 95th percentile value to the 5th percentile value.

Based on the analyst's (the author's) level of confidence in each source, a range factor and a probability (weight) have been assigned to each source in Table 1. The data reported by Bush has received a small range factor, relative to the other sources, because of the analyst's high level of confidence in this source. On the other hand, data reported in Rijnmond report was assigned a range factor of 9, indicating a lower level of confidence in this data source. The probability weights also represent the analyst's confidence in each data source.

Using the data given in Table 1, we have calculated prior distribution mean values (Table 2) using different method for comparison purposes. In this study we will use the mean values calculated using weighted average method.

Table 1
Pressure vessel failure mode generic data and assigned range factors and probabilities

Source	Disruptive (per year)	Non-disruptive (per year)	Assigned range factor	Weight factors[b]
Savannah River Site [13]	3.33E−04	3.24E−03	7	0.05
EEI-TVA [12]	3.00E−04	1.70E−03	4	0.08
EEI Boiler Drum [12]	1.40E−04	2.00E−04	4	0.08
Chemical [14]	5.48E−05	1.05E−04	3	0.11
UK Steam Drum Sample [12]	5.00E−05	6.00E−04	3	0.11
IRS-TUW [12]	4.50E−05	6.00E−04	3	0.11
NBBPV [12]	3.50E−05	–[a]	3	0.11
UK-Smith& Warwick [12]	3.20E−05	2.60E−04	4	0.08
CCPS [3]	9.55E−06	5.57E−05	5	0.07
German LWR Study Group [13]	8.80E−06	–[a]	4	0.08
ABMA [13]	4.20E−06	–[a]	4	0.08
Rijnmond [16]	1.00E−06	1.00E−05	9	0.04

[a] These sources have reported no frequencies for the non-disruptive.
[b] Weight factors are calculated using ranged factors and are normalized.

Table 2
Calculated prior distribution mean values for pressure vessel failure modes (alternate methods)

Method of calculation	Disruptive (per year)	Non-disruptive (per year)
Arithmetic mean (average)	8.40E−05	7.50E−04
Geometric mean	3.10E−05	4.10E−03
Weighted average[a]	**7.70E−05**	**4.80E−04**
Bayesian [7]	6.70E−05	6.00E−04

[a] Used in the case study.

Using the data given in Table 1, we have calculated prior distribution mean values (Table 2) using different method for comparison purposes. In this study we will use the mean values calculated using weighted average method.

3.2. Plant specific data

A hypothetical case is used to demonstrate the data analysis method presented in this paper. Consider company XYZ, a worldwide gas and oil firm, has collected pressure vessel failure data for their facilities for the past 15 years. The number of pressure vessels in operation is 187. The history has shown zero disruptive and seven non-disruptive failures for the past 15 years among the 187-pressure vessel population. Table 3 shows the evidence that will be used to update the generic data (prior).

3.3. Data updating

Using the constrained non-informative gamma distribution [15], the parameters of the prior distribution for the disruptive pressure vessel failure mode are calculated as follows:

$$\alpha_{\text{Prior}}^{\text{Dis}} = 0.5$$

$$\beta_{\text{Prior}}^{\text{Dis}} = 0.5/7.7E-05 = 6536$$

The posterior gamma distribution parameters are calculated:

$$\alpha_{\text{Post}}^{\text{Dis}} = 0 + 0.5 = 0.5$$

$$\beta_{\text{Post}}^{\text{Dis}} = 15 \times 187 + 6536 = 9341$$

The posterior mean is then calculated as:

$$\lambda_{\text{Post}}^{\text{Dis}} = 0.5/9341 = 5.4E-05$$

(events per year; disruptive failure mode)

Following the same steps for non-disruptive failure mode, we would get the following mean value for posterior distribution of the non-disruptive failure mode:

$$\lambda_{\text{Post}}^{\text{N-Dis}} = 2.0E-03$$

(events per year; non-disruptive failure mode)

Table 4 presents calculated distribution parameters of disruptive and non-disruptive pressure vessel failure modes for the example.

Table 3
Plant specific data for the case study

Poisson parameters	Disruptive	Non-disruptive
X (number of failures)	0	7
t (time interval)	15	15

Table 4
Prior and posterior parameters for pressure vessel failure modes

Gamma distribution parameters	Disruptive		Non-disruptive	
	Prior	Posterior	Prior	Posterior
α	0.5	0.5	0.5	7.5
β	6536	9341	1048	3853
λ (per year)	7.70E−05	5.40E−05	4.80E−04	2.00E−03
5th percentile (per year)	4.40E−05	3.10E−05	2.10E−05	9.20E−04
95th percentile (per year)	2.70E−04	1.90E−04	1.70E−03	3.20E−03

4. Conclusions

The method starts with a prior distribution, which must come from sources independent from the subject plant under study. An appropriate source is generic data reported in the literature or used in other studies. The issue with data reported in the literature is that it is mostly incomplete and often presented in terms of “point estimates”, which are single values representing the mean (or median) of the failure rate.

The depth of the evidence, e.g., plant data, is significant in shaping the characteristics of the posterior distribution. As shown in the case study, due to lack of disruptive failure at the plant the posterior mean (5.40E−05) is very close to that of the prior (7.70E−05). In comparison, the number of non-disruptive failure was relatively large, hence the difference between the prior and posterior means are almost an order of magnitude.

In conditions of insufficient data (incomplete prior knowledge) about the prior distribution or great uncertainty in the generic data sources, we may use “constrained non-informative priors” described by Atwood [17]. This representation of the prior preserves the mean value of the failure rate estimate and maintains a broad uncertainty range to accommodate the site-specific event data.

Although the methodology and the case study presented in this paper focus on the calculation of a time-based (i.e., failures per unit time) failure rate, based on a Poisson likelihood function and the conjugate gamma distribution, a similar method applies to the calculation of demand failure rates utilizing the binomial likelihood function and its conjugate beta distribution.

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